2103000206020032
EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (THIRD YEAR)(SIXTH SEMESTER)
MATHEMATICS –VII – LEVEL 2
(MTH-602-LINER ALGEBRA-II)

[**Time: As Per Schedule]** **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **BACHELOR OF SCIENCE (THIRD YEAR)(SIXTH SEMESTER)**
   b. Name of the Subject : **MATHEMATICS –VII – LEVEL 2 (MTH-602-LINER ALGEBRA-II)**
   c. Subject Code No : **2103000206020032**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

Student's Signature

Q.1 Answer the following questions (Any Five). 10

1) State existence theorem of linear map.

2) Is a linear map $T: V_3 \to V_2$; $T(x_1, x_2, x_3) = (x_1 - x_3, x_2 + x_3)$ be 1-1? Justify your answer.

3) Obtain the general rule of linear map $T: V_2 \to V_2$; $T(0,1) = (3,2)$ and $T(3,1) = (2,2)$.

4) Is a linear transformation $T: V_3 \to V_3$ defined by $T(x, y, z) = (y, y - z, x)$ be an Isomorphism? Justify.

5) Let $T: V_3 \to V_3$ be a one-one linear map then prove that $R(T) = V_3$.

6) Is $\begin{bmatrix} -2 & 2 \\ 3 & -2 \end{bmatrix}$ non singular? Justify your answer.

7) Prove that in an inner product space $V, u(\alpha v) = \bar{\alpha}(u \cdot v), \forall u, v \in V, a$ scalar α.

8) Write inner product of two vectors in n^{th} dimensional real and complex Inner product space.

Q.2 Answer the following. (Any Two) 10

(1) Let $T: U \to V$ be a linear map. Prove that If $v_1, v_2, v_3, \dots \dots, v_n$ are n L.I vectors of $R(T)$ and if $u_1, u_2, u_3, \dots \dots, u_n$ are n vectors of a vector space U with condition that $T(u_i) = v_i, \forall\, i = 1$ to n then $u_1, u_2, u_3, \dots \dots, u_n$ are L.I vectors of U.

(2) Let $T: U \to V$ be a linear transformation. Prove that $T(\alpha_1 u_1 + \alpha_2 u_2 + - - - + \alpha_n u_n) = \alpha_1 T(u_1) + \alpha_2 T(u_2) + - - - - + \alpha_n T(u_n)\cdot$ Where $\alpha_1, \alpha_2, , , , , , , , , , , , \alpha_n$ are scalars and $u_i's \in U$, $i = 1$ to n.

(3) Obtain the general rule for the given linear transformation $T: V_3 \to V_3$ defined by $T(e_1) = (1,2,1)$, $T(e_2) = (-1,3,1)$, $T(e_3) = (1, \frac{-1}{2}, -2)$. Where the set $\{e_1, e_2, e_3\}$ be a standard basis of a domain space V_3.

Q.3 Answer the following. (Any Two) 10

1) Is the linear transformation $S: V_3 \to V_3$ defined by $S(e_1) = e_1 + e_2$, $S(e_2) = e_1 - e_2 + e_3$ and $S(e_3) = 3e_1 + 4e_3$ non singular? Find its inverse if exists.

2) Prove that any vector space of dimension 4 is isomorphic to V_4.

3) Verify Rank -Nullity theorem for a linear transformation $S: V_3 \to V_3$ defined by $S(e_1) = e_1 - e_2$, $S(e_2) = e_2$ and $S(e_3) = e_1 + e_2 - 7e_3$.

Q.4 Answer the following. (Any Two) 10

1) Obtain the matrices $(T;\ B_1, B_2)$ associated with a linear transformation $T: V_3 \to V_3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, 2x_1 + 3x_2 - \frac{1}{2}x_3, x_1 + x_2 - 2x_3)$ relative to basis $B_1 = \{e_1, e_2, e_3\}$, $B_2 = \{(1,1,1), (1,2,3), (1,2,0)\}$ respective to domain and co domain vector space of linear map T.

2) Find Range, Rank, Kernel and Nullity of a matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$.

3) Find the Linear transformation T associated with a matrix $\begin{bmatrix} 2 & 1/2 & 3/2 \\ 1 & 3/2 & 1/2 \end{bmatrix}$ relative to basis $B_1 = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $B_2 = \{(1,1), (1,-1)\}$.

Q.5 Answer the following. (Any Two) 10

1) State and prove Schwarz's inequality in an Inner product space V.

2) Orthonormalized the L.I set $\{(1,1,1), (1,-1,1), (0,0,1)\}$ by Gram Schmidt's process.

3) In an Inner Product space V, Prove
   a) $\|\alpha u\| = |\alpha| \|u\|, \forall\, u \in V$ and α a scalar
   b) $\|u + v\| \leq \|u\| + \|v\|, \forall\, u, v \in V$.

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